Period 4, Feb 25, 2025

Graph the two following polar equations r_1 and r_2 in the same polar coordinate system. What is the relationship between the two graphs?

$r_1 = 3\cos 7\theta,\ r_2 = 3\cos 7\underbrace{\left(\theta - \frac{\pi}{4}\right)}_{=0}$

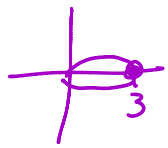
New Start

$\theta - \frac{\pi}{4} = 0 \Rightarrow \theta = \frac{\pi}{4}$

7 Petals

Rotation $\frac{\pi}{4}$

θ	r
0	$3 = 3\cos 0 = 3 \cdot 1$

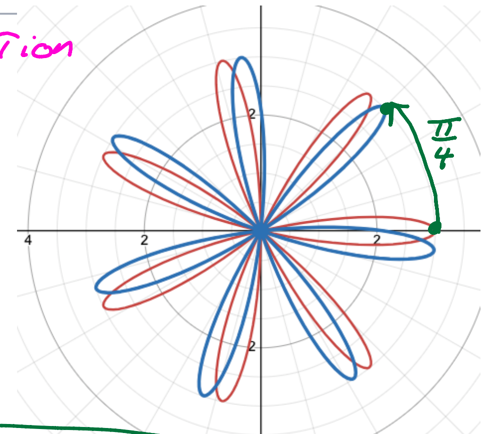


$r = 3\cos 7\theta$

$0 \le \theta \le \pi$

$r = 3\cos 7\left(\theta - \frac{\pi}{4}\right)$

$0 \le \theta \le \pi$



Solve the equation $x^3 - 5x^2 + 2x + 8 = 0$ given that -1 is a zero of $f(x) = x^3 - 5x^2 + 2x + 8$.

$1x^3 - 5x^2 + 2x + 8$

$\frac{x^3 - 5x^2 + 2x + 8}{x+1} = 1x^2 - 6x + 8$

$$\begin{array}{r|rrrr} -1 & 1 & -5 & 2 & 8 \\ & & -1 & 6 & -8 \\ \hline & 1 & -6 & 8 & \boxed{0} \end{array}$$

$x^3 - 5x^2 + 2x + 8 = (x+1)(\underbrace{x^2 - 6x + 8})$

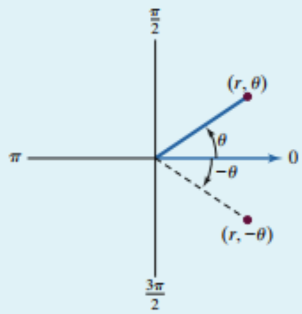
$x^3 - 5x^2 + 2x + 8 = (x+1)(x-4)(x-2) = 0$

$0 = x+1$ or $0 = x-4$ or $0 = x-2$

$x = -1$ or $x = 4$ or $x = 2$

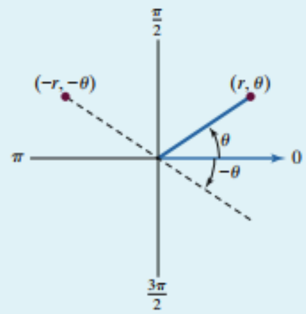
Tests for Symmetry in Polar Coordinates

Symmetry with Respect to the Polar Axis (x-Axis)



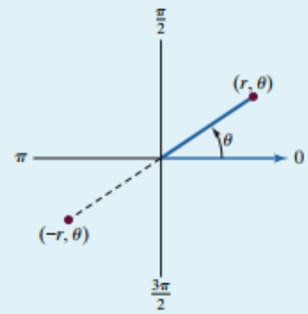
Replace θ with $-\theta$. If an equivalent equation results, the graph is symmetric with respect to the polar axis.

Symmetry with Respect to the Line $\theta = \frac{\pi}{2}$ (y-Axis)



Replace (r, θ) with $(-r, -\theta)$. If an equivalent equation results, the graph is symmetric with respect to $\theta = \frac{\pi}{2}$.

Symmetry with Respect to the Pole (Origin)



Replace r with $-r$. If an equivalent equation results, the graph is symmetric with respect to the pole.

Test for symmetry and then graph the polar equation.

$r = 5 + 2\,\mathbf{sin}\,2\theta$

Test For Polar axis

$\left.\begin{array}{l} r = 5+2\sin 2(-\theta) \\ r = 5+2\sin(-2\theta) \end{array}\right\}$ odd

$r = 5 - 2\sin 2\theta$

Nope

↓

May not be Symmetric

Test For $\theta = \frac{\pi}{2}$

$r = 5 + 2\sin 2\theta$

$-r = 5+2\sin 2(-\theta)$

$-1 \cdot -r = (5-2\sin 2\theta)\cdot -1$

$r = -5+2\sin 2\theta$

Nope

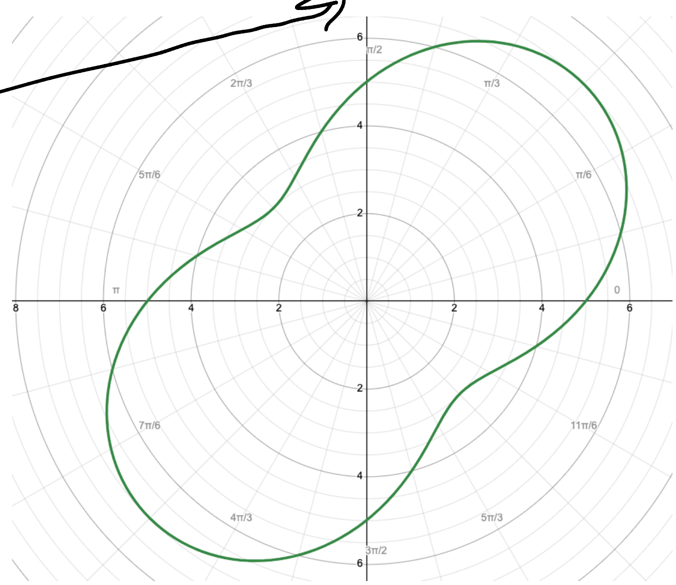
origin Test

$r = 5 + 2\sin 2\theta$

$-1 \cdot -r = (5+2\sin 2\theta)\cdot -1$

$+r = -5-2\sin 2\theta$

Nope



Test for symmetry and graph the polar equation.

$r^2 = 9\,\mathbf{cos}\,(2\theta)$

Test For Polar axis

$r^2 = 9\cos 2\theta$

$r^2 = 9\cos 2(-\theta)$

$r^2 = 9\cos(-2\theta)$

Even

$r^2 = 9\cos 2\theta$

Same

Yes

Test For $\theta = \frac{\pi}{2}$

$r^2 = 9\cos 2\theta$

$(-r)^2 = 9\cos 2(-\theta)$

$r^2 = 9\cos 2\theta$

Same

Yes

Origin Test

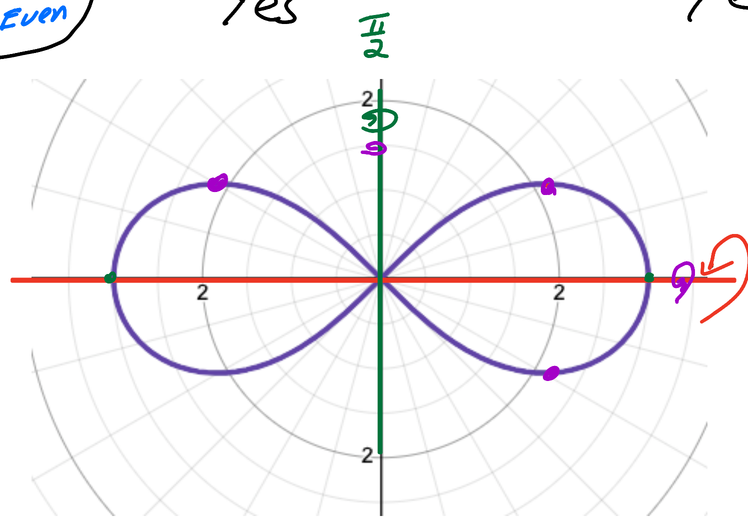
$r^2 = 9\cos 2\theta$

$(-r)^2 = 9\cos 2\theta$

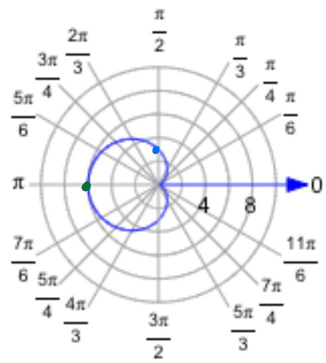
$r^2 = 9\cos 2\theta$

Same

Yes



The graph of a polar equation is given. Select the polar equation for the graph.



Choose the correct equation below.

- $r = 3 - 3\cos\theta$
- $r = 3\cos\theta$
- $r = 3\sin\theta$
- $r = 3 - 3\sin\theta$

$\theta = \frac{\pi}{2}$

r is close To 3

$3 - 3\cos\frac{\pi}{2} = 3 - 3 \cdot 0 = 3 - 0 = 3$

$\cancel{3\cos\frac{\pi}{2} = 3 \cdot 0 = 0}$

$3\sin\frac{\pi}{2} = 3 \cdot 1 = 3$

$\cancel{3 - 3\sin\frac{\pi}{2} = 3 - 3 \cdot 1 = 3 - 3 = 0}$

$\theta = \pi$

$r = 6$

$3 - 3\cos\pi = 3 - 3(-1)$
$3 + 3 = 6$

$\cancel{3\sin\pi = 3 \cdot 0 = 0}$

Rewrite the following expression in terms of the given function.

$$\frac{\cot x}{\tan x + \cot x};\ \sec x = \frac{1}{\cos x} \Rightarrow \frac{1}{\sec x} = \cos x$$

$$\frac{\frac{\cos x}{\sin x}}{\frac{\sin x \sin x}{\cos x \sin x} + \frac{\cos x \cos x}{\sin x \cos x}} = \frac{\frac{\cos x}{\sin x}}{\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}} = \frac{\frac{\cos x}{\sin x}}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}} = \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x \cos x}}$$

$$\frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}\cos x}{1} = \frac{\cos^2 x}{1} = \left(\frac{1}{\sec x}\right)^2 = \frac{1}{\sec^2 x}$$

Find the exact value of the following expression.

$$\tan\left(\frac{\pi}{6}+\frac{5\pi}{4}\right) = \frac{\tan\frac{\pi}{6}+\tan\frac{5\pi}{4}}{1-\tan\frac{\pi}{6}\cdot\tan\frac{5\pi}{4}}$$

$$\tan(a+b) = \frac{\tan a+\tan b}{1-\tan a\tan b}$$

$$\tan\frac{\pi}{6} = \frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\tan\frac{5\pi}{4} = \frac{\sin\frac{5\pi}{4}}{\cos\frac{5\pi}{4}} = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$$

$$\frac{\frac{1}{\sqrt{3}} + \frac{1\sqrt{3}}{1\cdot\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}}\cdot 1 - \frac{1}{\sqrt{3}}\cdot\frac{1}{1}} = \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{1+\sqrt{3}}{\cancel{\sqrt{3}}}\cdot\frac{\cancel{\sqrt{3}}}{\sqrt{3}-1}$$

$$\frac{(1+\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{3}+1+3+\sqrt{3}}{3+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

$$\frac{4}{2}+\frac{2\sqrt{3}}{2}$$

Solve the following equation on the interval [0,2π).

around Twice

$$\cos\left(2x-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$2x-\frac{\pi}{4} = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4} \Rightarrow \frac{2x}{2} = \frac{2\pi}{2\cdot 4}, \frac{8\pi}{2\cdot 4}, \frac{10\pi}{2\cdot 4}, \frac{16\pi}{2\cdot 4}$$

$$+\frac{\pi}{4} \quad +\frac{\pi}{4} \quad +\frac{\pi}{4} \quad +\frac{\pi}{4} \quad +\frac{\pi}{4}$$

$$x = \frac{2\pi}{8}, \frac{8\pi}{8}, \frac{10\pi}{8}, \frac{16\pi}{8} = \frac{\pi}{4}, \pi, \frac{5\pi}{4}, \cancel{2\pi}$$

Use substitution to determine whether the given x-value is a solution of the equation.

$$\cos x = \sin 2x,\ x = \frac{\pi}{2}$$

$$\cos\frac{\pi}{2} = \sin\left(2\left(\frac{\pi}{2}\right)\right)$$

$$\cos\frac{\pi}{2} = \sin\pi$$

$$0 = 0$$

yes!!!

Solve the equation on the interval $[0,2\pi)$.

$$3\sin x + 6\sqrt{2} = \sin x + 5\sqrt{2}$$
$$-\sin x \quad -6\sqrt{2} \qquad -\sin x \quad -6\sqrt{2}$$

$$\frac{\cancel{2}\sin x}{\cancel{2}} = \frac{-\sqrt{2}}{2}$$

$$\sin x = -\frac{\sqrt{2}}{2} \Rightarrow x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

Use the given information to find the exact value of each of the following.

a. **sin** 2θ b. **cos** 2θ c. **tan** 2θ

cot $\theta = 12$, θ lies in quadrant III

$\sin\theta = -$

$\cos\theta = -$

$$\cot^2\theta + 1 = \csc^2\theta$$
$$12^2 + 1 = \csc^2\theta$$
$$144 + 1 = \csc^2\theta$$
$$145 = \csc^2\theta = \frac{1}{\sin^2\theta}$$

$$\sin\theta = \pm\frac{1}{\sqrt{145}}$$

$$\sin\theta = -\frac{1}{\sqrt{145}}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(\frac{-1}{\sqrt{145}}\right)^2 + \cos^2\theta = 1$$
$$\frac{1}{145} + \cos^2\theta = 1 - \frac{1}{145}$$
$$-\frac{1}{145}$$
$$\cos^2\theta = \frac{145}{145} - \frac{1}{145}$$
$$\sqrt{\cos^2\theta} = \sqrt{\frac{144}{145}}$$
$$\cos\theta = \pm\frac{12}{\sqrt{145}}$$
$$\cos\theta = \frac{-12}{\sqrt{145}}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\sin 2\theta = 2 \cdot \frac{-1}{\sqrt{145}} \cdot \frac{-12}{\sqrt{145}} = \frac{24}{145}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{144}{145} - \frac{1}{145} = \frac{143}{145}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{24}{145}}{\frac{143}{145}} = \frac{24}{\cancel{145}} \cdot \frac{\cancel{145}}{143} = \frac{24}{143}$$

Verify the identity.

$$(\sec x - \tan x)^2 = \frac{1 - \sin x}{1 + \sin x}$$

To verify the identity, start with the more complicated side and transform it to look like the other si

$(\sec x - \tan x)^2$

$$= \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2$$

Rewrite in terms of sine and cosine inside parentheses.

(Do not simplify.)

$$\frac{(1-\sin x)^2}{\cos^2 x}$$

$$1-\sin^2 x = 1^2 - \sin^2 x$$

$$(1-\sin x)(1+\sin x)$$

$$= \frac{1 - 2\sin x + \sin^2 x}{\cos^2 x}$$

Combine and square.

(Simplify your answer.)

$$= \frac{1 - 2\sin x + \sin^2 x}{1 - \sin^2 x}$$

Rewrite the denominator by applying a Pythagorean identity.

(Do not factor.)

$\sin x \neq 1$ $\quad$ $x \neq \frac{\pi}{2}$

$$= \frac{(1 - \cancel{\sin x})(1 - \sin x)}{(1 - \cancel{\sin x})(1 + \sin x)}$$

Factor the numerator and the denominator.

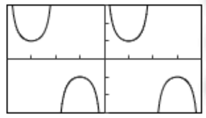
(Factor completely. Do not simplify.)

$$= \frac{1 - \sin x}{1 + \sin x}$$

Divide out the common factor.

To the right, half of an identity and the graph of this half are given. Use the graph to make a conjecture as to what the right side of the identity should be. Then prove your conjecture. The viewing window is [−2π, 2π, π/2] by [−3, 3, 1].

$$\frac{\sin(2x)}{\cos x} + \frac{\cos(2x)}{\sin x} = ?$$



$$\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} = \frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}} + \frac{1-2\sin^2 x}{\sin x}$$

$$2\sin x + \frac{1}{\sin x} - \frac{2\sin^{\cancel{2}} x}{\cancel{\sin x}}$$

$$\cancel{2\sin x} + \frac{1}{\sin x} - \cancel{2\sin} x$$

Which of the following completes the above identity?

- − csc x
- csc x ✓
- sec x
- − sec x

$$\frac{1}{\sin x} = \csc x$$

Which of the following proves the conjectured identity?

- $\dfrac{-\cos^3 x - \sin^2 x \cos x}{\sin x \cos x}$
- $\dfrac{-\cos^2 x \sin x - \sin^3 x}{\sin x \cos x} = \dfrac{-\cancel{\sin x}(\cos^2 x + \sin^2 x)}{\cancel{\sin x}\cos x} = \dfrac{-1}{\cos x}$

Nope

- $\dfrac{\sin^2 x \cos x + \cos^3 x}{\sin x \cos x}$ ✓

$$\frac{\cancel{\cos x}(\sin^2 x + \cos^2 x)}{\sin x \cancel{\cos x}} = \frac{\sin^2 x \cos x + \cos^3 x}{\sin x \cos x} = \frac{1}{\sin x}$$

- $\dfrac{\sin x \cos^2 x + \sin^3 x}{\sin x \cos x}$